INTRODUCTION TO STRUCTURE FUNCTIONS [1]

INTRODUCTION AUX FONCTIONS DE STRUCTURE

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Abstract

The theory of deep inelastic scattering structure functions is reviewed with an emphasis put on the QCD expectations of their behaviour in the region of small values of the Bjorken parameter x.

La théorie des fonctions de structure de diffusion profondement inélastique est présentée avec une attention particulière consacrée aux prévisions de la Chromodynamique Quantique de leur comportement dans la région des petites valeurs du paramètre x de Bjorken.

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The advent of the HERA ep collider has opened up a possibility to test QCD in the new and hitherto unexplored regime of the small values of the Bjorken parameter x. This parameter is, as usual, defined as $x = Q^2/(2pq)$ where p is the proton four momentum, q the four momentum transfer between the leptons and $Q^2 = -q^2$. Perturbative QCD predicts that several new phenomena will occur when the parameter x specifying the longitudinal momentum fraction of a hadron carried by a parton (i.e. by a quark or by a gluon) becomes very small [1, 2] . The main expectation is that the gluon densities should strongly grow in this limit, eventually leading to the parton saturation effects [1, 2, 3, 4]. The small x behaviour of the structure functions is driven by the gluon through the $g \to q\bar{q}$ transition and the increase of gluon distributions with decreasing x implies a similar increase of the deep inelastic lepton - proton scattering structure function F_2 as the Bjorken parameter x decreases [7]. The recent experimental data are consistent with this perturbative QCD prediction that the structure function $F_2(x, Q^2)$ should strongly grow with the decreasing Bjorken parameter x [8, 9, 10].

The purpose of this talk is to summarize the QCD expectations for the small x behaviour of the deep inelastic scattering structure functions. After briefly reviewing predictions of the Regge theory we shall discuss the Balitzkij, Lipatov, Fadin, Kuraev (BFKL) equation which sums the leading powers of $\alpha_s ln(1/x)$. We will also briefly describe the "conventional" formalism based on the leading (and next to leading) order QCD evolution equations and confront it with the BFKL equation. Besides the structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$ we shall also consider the spin structure function $g_1(x, Q^2)$. The novel feature in the latter case is the appearence of the double logarithmic terms i.e. powers of $\alpha_s ln^2(1/x)$ at each order of the perturbative expansion.

Small x behaviour of structure functions for fixed Q^2 reflects the high energy behaviour of the virtual Compton scattering total cross-section with increasing total CM energy squared W^2 since $W^2 = Q^2(1/x - 1)$. The appropriate framework for the theoretical description of this behaviour is the Regge pole exchange picture [12] .

The high energy behaviour of the total hadronic and (real) photoproduction cross-sections can be economically described by two contributions: an (effective) pomeron with its intercept slightly above unity (~ 1.08) and the leading meson Regge trajectories with intercept $\alpha_R(0) \approx 0.5$ [11]. The reggeons can be identified as corresponding to ρ, ω, f or A_2 exchange(s) depending upon the quantum numbers involved. All these reggeons have approximately the same intercept. One refers to the pomeron obtained from the phenomenological analysis of hadronic total cross sections as the "soft" pomeron since the bulk of the processes building-up the cross sections are low p_t (soft) processes.

The Regge pole model gives the following parametrization of the deep inelastic scattering structure function $F_2(x, Q^2)$ at small x

$$F_2(x, Q^2) = \sum_i \tilde{\beta}_i(Q^2) x^{1-\alpha_i(0)}. \tag{1}$$

The relevant reggeons are those which can couple to two (virtual) photons. The (singlet) part of the structure function F_2 is controlled at small x by pomeron exchange, while the non-singlet part $F_2^{NS} = F_2^p - F_2^n$ by the A_2 reggeon. Neither pomeron nor A_2 reggeons couple to the spin structure function $g_1(x, Q^2)$ which is described at small x by the exchange of reggeons corresponding to axial vector mesons [13, 14] i.e. to A_1 exchange for the non-singlet part

$g_1^{NS} = g_1^p - g_1^n$ etc.

$$g_1^{NS}(x, Q^2) = \gamma(Q^2) x^{-\alpha_{A_1}(0)}. \tag{2}$$

The reggeons which correspond to axial vector mesons are expected to have very low intercept (i.e. $\alpha_{A_1} \leq 0$ etc.).

At small x the dominant role is played by the gluons and the basic dynamical quantity is the unintegrated gluon distribution $f(x, Q_t^2)$ where x denotes the momentum fraction of a parent hadron carried by a gluon and Q_t its transverse momentum. The unintegrated distribution $f(x, Q_t^2)$ is related in the following way to the more familiar scale dependent gluon distribution $g(x, Q^2)$:

$$xg(x, Q^2) = \int^{Q^2} \frac{dQ_t^2}{Q_t^2} f(x, Q_t^2). \tag{3}$$

In the leading $ln(1/x)$ approximation the unintegrated distribution $f(x, Q_t^2)$ satisfies the BFKL equation [5, 6, 18] which has the following form:

$$f(x, Q_t^2) = f^0(x, Q_t^2) +$$

$$\bar{\alpha}_s \int_x^1 \frac{dx'}{x'} \int \frac{d^2q}{\pi q^2} \left[\frac{Q_t^2}{(\boldsymbol{q} + \boldsymbol{Q_t})^2} f(x', (\boldsymbol{q} + \boldsymbol{Q_t})^2) - f(x', Q_t^2) \Theta(Q_t^2 - q^2) \right] \tag{4}$$

where

$$\bar{\alpha}_s = \frac{3\alpha_s}{\pi} \tag{5}$$

This equation sums the ladder diagrams with gluon exchange accompanied by virtual corrections which are responsible for the gluon reggeization. The first and the second terms on the right hand side of eq. (4) correspond to real gluon emission with q being the transverse momentum of the emitted gluon, and to the virtual corrections respectively. $f^0(x, Q_t^2)$ is a suitably defined inhomogeneous term.

For the fixed coupling case eq. (4) can be solved analytically and the leading behaviour of its solution at small x is given by the following expression:

$$f(x, Q_t^2) \sim (Q_t^2)^{\frac{1}{2}} \frac{x^{-\lambda_{BFKL}}}{\sqrt{ln(\frac{1}{x})}} exp\left(-\frac{ln^2(Q_t^2/\bar{Q}^2)}{2\lambda" ln(1/x)} \right) \tag{6}$$

with

$$\lambda_{BFKL} = 4ln(2)\bar{\alpha}_s \tag{7}$$

$$\lambda" = \bar{\alpha}_s 28\zeta(3) \tag{8}$$

where the Riemann zeta function $\zeta(3) \approx 1.202$. The parameter $\bar{Q}$ is of nonperturbative origin.

The quantity $1 + \lambda_{BFKL}$ is equal to the intercept of the so - called BFKL pomeron. Its potentially large magnitude (~ 1.5) should be contrasted with the intercept $\alpha_{soft} \approx 1.08$ of the (effective) "soft" pomeron which has been determined from the phenomenological analysis of the high energy behaviour of hadronic and photoproduction total cross-sections [11].

The solution of the BFKL equation reflects its diffusion pattern which is the direct consequence of the absence of transverse momentum ordering along the gluon chain. The interrelation between the diffusion of transverse momenta towards both the infrared and ultraviolet regions **and** the increase of gluon distributions with decreasing x is a characteristic property of QCD

at low x. It has important consequences for the structure of the hadronic final state in deep inelastic scattering at small x [4, 15, 16].

In practice one introduces the running coupling $\bar{\alpha}_s(Q_t^2)$ in the BFKL equation (4). This requires the introduction of an infrared cut-off to prevent entering the infrared region where the coupling becomes large. The effective intercept λ_{BFKL} found by numerically solving the equation depends on the magnitude of this cut-off [29]. The impact of the momentum cut-offs on the solution of the BFKL equation has also been discussed in refs. [30, 31]. In impact parameter representation the BFKL equation offers an interesting interpretation in terms of colour dipoles [32]. The application of this formalism to the phenomenological analysis of deep inelastic scattering is presented in a talk given by Samuel Wallon [33]. It should also be emphasised that the complete calculation of the next-to-leading corrections to the BFKL equation has recently become presented in ref. [34].

The structure functions $F_{2,L}(x,Q^2)$ are driven at small x by the gluons and are related in the following way to the unintegrated distribution f:

$$F_{2,L}(x,Q^2) = \int_x^1 \frac{dx'}{x'} \int \frac{dQ_t^2}{Q_t^2} F_{2,L}^{box}(x', Q_t^2, Q^2) f(\frac{x}{x'}, Q_t^2). \tag{9}$$

The functions $F_{2,L}^{box}(x', Q_t^2, Q^2)$ may be regarded as the structure functions of the off-shell gluons with virtuality Q_t^2. They are described by the quark box (and crossed box) diagram contributions to the photon-gluon interaction. The small x behaviour of the structure functions reflects the small z ($z = x/x'$) behaviour of the gluon distribution $f(z, Q_t^2)$.

Equation (9) is an example of the "k_t factorization theorem" which relates measurable quantities (like DIS structure functions) to the convolution in both longitudinal as well as in transverse momenta of the universal gluon distribution $f(z, Q_t^2)$ with the cross-section (or structure function) describing the interaction of the "off-shell" gluon with the hard probe [25, 26]. The k_t factorization theorem is the basic tool for calculating the observable quantities in the small x region in terms of the (unintegrated) gluon distribution f which is the solution of the BFKL equation.

The leading - twist part of the k_t factorization formula can be rewritten in a collinear factorization form. The leading small x effects are then automatically resummed in the splitting functions and in the coefficient functions. The k_t factorization theorem can in fact be used as the tool for calculating these quantities. Thus, for instance, the moment function $\bar{P}_{qg}(\omega, \alpha_s)$ of the splitting $P_{qg}(z, \alpha_s)$ function is represented in the following form (in the so called Q_0^2 regularization and DIS scheme [26]):

$$\bar{P}_{qg}(\omega, \alpha_s) = \frac{\gamma_{gg}^2(\frac{\bar{\alpha}_s}{\omega}) \tilde{F}_2^{box}\left(\omega = 0, \gamma = \gamma_{gg}(\frac{\bar{\alpha}_s}{\omega})\right)}{2\sum_i e_i^2} \tag{10}$$

where $\tilde{F}_2^{box}(\omega, \gamma)$ is the Mellin transform of the moment function $\bar{F}_2^{box}(\omega, Q_t^2, Q^2)$ i.e.

$$\bar{F}_2^{box}(\omega, Q_t^2, Q^2) = \frac{1}{2\pi i} \int_{1/2-i\infty}^{1/2+i\infty} d\gamma \tilde{F}_2^{box}(\omega, \gamma) \left(\frac{Q^2}{Q_t^2}\right)^{\gamma} \tag{11}$$

and the anomalous dimension $\gamma_{gg}(\frac{\bar{\alpha}_s}{\omega})$ has the following expansion [28];

$$\gamma_{gg}(\frac{\bar{\alpha}_s}{\omega}) = \sum_{n=1}^{\infty} c_n \left(\frac{\bar{\alpha}_s}{\omega}\right)^n \tag{12}$$

This expansion gives the following expansion of the splitting function P_{gg}

$$zP_{gg}(z,\alpha_s)=\sum_{1}^{\infty}c_n\frac{[\alpha_s ln(1/z)]^{n-1}}{(n-1)!} \tag{13}$$

Representation (10) generates the following expansion of the splitting function $P_{qg}(z,\alpha_s)$ at small z:

$$zP_{qg}(z,\alpha_s)=\frac{\alpha_s}{2\pi}zP^{(0)}(z)+(\bar{\alpha}_s)^2\sum_{n=1}^{\infty}b_n\frac{[\bar{\alpha}_s ln(1/z)]^{n-1}}{(n-1)!} \tag{14}$$

The first term on the right hand side of eq. (14) vanishes at $z=0$. It should be noted that the splitting function P_{qg} is formally non-leading at small z when compared with the splitting function P_{gg} . For moderately small values of z however, when the first few terms in the expansions (12) and (14) dominate, the BFKL effects can be much more important in P_{qg} than in P_{gg}. This comes from the fact that in the expansion (14) all coefficients b_n are different from zero while in eq. (12) we have $c_2=c_3=0$ [28]. The small x resummation effects within the conventional QCD evolution formalism have recently been discussed in refs. [35, 36, 37, 38]. One finds in general that at the moderately small values of x which are relevant for the HERA measurements, the small x resummation effects in the splitting function P_{qg} have a much stronger impact on F_2 than the small x resummation in the splitting function P_{gg}. This reflects the fact, which has already been mentioned above, that in the expansion (14) all coefficients b_n are different from zero while in eq. (12) we have $c_2=c_3=0$. It should also be remembered that the BFKL effects in the splitting function $P_{qg}(z,\alpha_s)$ can significantly affect extraction of the gluon distribution out of the experimental data on the slope of the structure function $F_2(x,Q^2)$ which is based on the following relation:

$$Q^2\frac{\partial F_2(x,Q^2)}{\partial Q^2}\simeq 2\sum_i e_i^2\int_x^1 dzP_{qg}(z,\alpha_s(Q^2))\frac{x}{z}g(\frac{x}{z},Q^2) \tag{15}$$

A more general treatment of the gluon ladder than that which follows from the BFKL formalism is provided by the Catani, Ciafaloni, Fiorani, Marchesini (CCFM) equation based on angular ordering along the gluon chain [19, 20]. This equation embodies both the BFKL equation at small x and the conventional Altarelli-Parisi evolution at large x. The unintegrated gluon distribution f now acquires dependence upon an additional scale Q which specifies the maximal angle of gluon emission. The CCFM equation has the following form :

$$\begin{aligned} f(x,Q_t^2,Q^2) &= \hat{f}^0(x,Q_t^2,Q^2)+ \\ \bar{\alpha}_s\int_x^1\frac{dx'}{x'}\int\frac{d^2q}{\pi q^2}\Theta(Q-qx/x')\Delta_R(\frac{x}{x'},Q_t^2,q^2)&\frac{Q_t^2}{(\boldsymbol{q}+\boldsymbol{Q_t})^2}f(x',(\boldsymbol{q}+\boldsymbol{Q_t})^2,q^2)) \end{aligned} \tag{16}$$

where the theta function $\Theta(Q-qx/x')$ reflects the angular ordering constraint on the emitted gluon. The "non-Sudakov" form-factor $\Delta_R(z,Q_t^2,q^2)$ is now given by the following formula:

$$\Delta_R(z,Q_t^2,q^2)=exp\left[-\bar{\alpha}_s\int_z^1\frac{dz'}{z'}\int\frac{dq'^2}{q'^2}\Theta(q'^2-(qz')^2)\Theta(Q_t^2-q'^2)\right] \tag{17}$$

Eq.(16) still contains only the singular term of the $g\to gg$ splitting function at small z. Its generalization which would include remaining parts of this vertex (as well as quarks) is possible. The numerical analysis of this equation was presented in ref. [20] . The CCFM equation which is the generalization of the BFKL equation generates the steep $x^{-\lambda}$ type of behaviour for the deep inelastic structure functions as the effect of the leading $ln(1/x)$ resummation [39]. The

slope λ turns out to be sensitive on the (formally non-leading) additional constraint $q^2 < Q_t^2 x'/x$ in eq. (16) which follows from the requirement that the virtuality of the last gluon in the chain is dominated by Q_t^2 [22, 23].

The HERA data can be described quite well using the BFKL and CCFM equations combined with the factorization formula (9) [7, 39, 23]. One can however obtain satisfactory description of the HERA data staying within the scheme based on the Altarelli-Parisi equations alone without the small x resummation effects being included in the formalism [40, 41]. In the latter case the singular small x behaviour of the gluon and sea quark distributions has to be introduced in the parametrization of the starting distributions at the moderately large reference scale $Q^2 = Q_0^2$ (i.e. $Q_0^2 \approx 4GeV^2$ or so) [40]. One can also generate steep behaviour dynamically starting from non-singular "valence-like" parton distributions at some very low scale $Q_0^2 = 0.35GeV^2$ [41]. In the latter case the gluon and sea quark distributions exhibit "double logarithmic behaviour" [42]

$$F_2(x, Q^2) \sim exp\left(2\sqrt{\xi(Q^2, Q_0^2) ln(1/x)}\right) \tag{18}$$

where

$$\xi(Q^2, Q_0^2) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{3\alpha_s(q^2)}{\pi}. \tag{19}$$

For very small values of the scale Q_0^2 the evolution length $\xi(Q^2, Q_0^2)$ can become large for moderate and large values of Q^2 and the "double logarithmic" behaviour (18) is, within the limited region of x, similar to that corresponding to the power like increase of the type $x^{-\lambda}$, $\lambda \approx 0.3$.

The discussion presented above concerned the small x behaviour of the singlet structure function which was driven by the gluon through the $g \to q\bar{q}$ transition. The gluons of course decouple from the non-singlet channel and the mechanism of generating the small x behaviour in this case is different.

The novel feature of the non-singlet channel is the appearence of the **double** logarithmic terms i.e. powers of $\alpha_s ln^2(1/x)$ at each order of the perturbative expansion [43, 44, 45, 46, 47]. These double logarithmic terms are generated by the ladder diagrams with quark (antiquark) exchange along the chain. The ladder diagrams can acquire corrections from the "bremsstrahlung" contributions [45, 47] which do not vanish for the polarized structure function $g_1^{NS}(x, Q^2)$ [47]. They are however relatively unimportant and are non-leading in the $1/N_c$ expansion.

In the approximation where the leading double logarithmic terms are generated by ladder diagrams with quark (antiquark) exchange along the chain the unintegrated non-singlet spin dependent quark distribution $\Delta f_q^{NS}(x, k_t^2)$ ($\Delta f_q^{NS} = \Delta u + \Delta\bar{u} - \Delta d - \bar{\Delta} d$) satisfies the following integral equation :

$$\Delta f_q^{NS}(x, Q_t^2) = \Delta f_{q0}^{NS}(x, Q_t^2) + \tilde{\alpha}_s \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{\frac{Q_t^2}{z}} \frac{dQ_t'^2}{Q_t'^2} \Delta f_q^{NS}(\frac{x}{z}, Q_t'^2) \tag{20}$$

where

$$\tilde{\alpha}_s = \frac{2}{3\pi}\alpha_s \tag{21}$$

and Q_0^2 is the infrared cut-off parameter. The upper limit Q_t^2/z in the integral equation (20) follows from the requirement that the virtuality of the quark at the end of the chain is dominated

by Q_t^2. A possible non-perturbative A_1 reggeon contribution has to be introduced in the driving term i.e.

$$\Delta f_{q0}^{NS}(x, Q_t^2) \sim x^{-\alpha_{A_1}(0)} \tag{22}$$

at small x.

Equation (20) implies the following equation for the moment function $\Delta \bar{f}_q^{NS}(\omega, Q_t^2)$

$$\Delta \bar{f}_q^{NS}(\omega, Q_t^2) = \Delta \bar{f}_{q0}^{NS}(\omega, Q_t^2) + \frac{\tilde{\alpha}_s}{\omega}\left[\int_{Q_0^2}^{Q_t^2} \frac{dQ_t'^2}{Q_t'^2} \Delta \bar{f}_q^{NS}(\omega, Q_t'^2) + \int_{Q_t^2}^{\infty} \frac{dQ_t'^2}{Q_t'^2}\left(\frac{Q_t^2}{Q_t'^2}\right)^{\omega} \Delta \bar{f}_q^{NS}(\omega, Q_t'^2)\right] \tag{23}$$

For fixed coupling $\tilde{\alpha}_s$ equation (23) can be solved analytically. Assuming for simplicity that the inhomogeneous term is independent of Q_t^2 (i.e. that $\Delta \bar{f}_{q0}^{NS}(\omega, Q_t^2) = C(\omega)$) we get the following solution of eq.(23):

$$\Delta \bar{f}_q^{NS}(\omega, Q_t^2) = C(\omega) R(\tilde{\alpha}_s, \omega) \left(\frac{Q_t^2}{Q_0^2}\right)^{\gamma^-(\tilde{\alpha}_s, \omega)} \tag{24}$$

where

$$\gamma^-(\tilde{\alpha}_s, \omega) = \frac{\omega - \sqrt{\omega^2 - 4\tilde{\alpha}_s}}{2} \tag{25}$$

and

$$R(\tilde{\alpha}_s, \omega) = \frac{\omega \gamma^-(\tilde{\alpha}_s, \omega)}{\tilde{\alpha}_s}. \tag{26}$$

Equation (25) defines the anomalous dimension of the moment of the non-singlet quark distribution in which the double logarithmic $ln(1/x)$ terms i.e. the powers of $\frac{\alpha_s}{\omega^2}$ have been resummed to all orders. It can be seen from (25) that this anomalous dimension has a (square root) branch point singularity at $\omega = \bar{\omega}$ where

$$\bar{\omega} = 2\sqrt{\tilde{\alpha}_s}. \tag{27}$$

This singularity will of course be also present in the moment function $\Delta \bar{f}_q^{NS}(\omega, Q_t^2)$ itself. It should be noted that in contrast to the BFKL singularity whose position above unity was proportional to α_s, $\bar{\omega}$ is proportional to $\sqrt{\alpha_s}$ - this being the straightforward consequence of the fact that equation (23) sums double logarithmic terms $(\frac{\alpha_s}{\omega^2})^n$. This singularity gives the following contribution to the non-singlet quark distribution $\Delta f_q^{NS}(x, Q_t^2)$ at small x:

$$\Delta f_q^{NS}(x, Q_t^2) \sim \frac{x^{-\bar{\omega}}}{ln^{3/2}(1/x)}. \tag{28}$$

The introduction of the running coupling effects in equation (23) turns the branch point singularity into the series of poles which accumulate at $\omega = 0$ [44]. The numerical analysis of the corresponding integral equation, with the running coupling effects taken into account, gives an effective slope ,

$$\lambda(x, Q_t^2) = \frac{dln \Delta f_q^{NS}(x, Q_t^2)}{dln(1/x)} \tag{29}$$

with magnitude $\lambda(x, Q_t^2) \approx 0.2 - 0.3$ at small x [48]. The result of this estimate suggest that a reasonable extrapolation of the (non-singlet) polarized quark densities would be to assume an $x^{-\lambda}$ behaviour with $\lambda \approx 0.2 - 0.3$. Similar extrapolations of the spin-dependent quark distributions towards the small x region have been assumed in several recent parametrizations of parton densities [49, 50, 51, 52]. The perturbative QCD effects become significantly amplified for the

singlet spin structure function due to mixing with the gluons. The simple ladder equation may not however be applicable for an accurate description of the double logarithmic terms in the polarized gluon distribution ΔG [53]. The small x behaviour of the spin dependent structure function g_1 has also been disscussed in refs. [54, 55].

To sum up we have briefly summarised the theoretical QCD expectations for the structure functions at low x. We have limited ourselves to the region of large Q^2 where perturbative QCD becomes applicable. Specific problems of the low Q^2, low x region are discussed in ref. [56].

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